Calculus Warm Up # 8 -1

1) Evaluate:

$$\int \frac{e^{3x}}{(1+e^{3x})^2}\,dx$$

2) evaluate

$$\frac{d}{dx}\left[\frac{e^{x^2}}{x}\right]$$

HW Questions: p. 370

In Exercises 43–62, evaluate the integral.

43. $\int_0^1 e^{-2x}\,dx$ $\rightarrow$ $u = -2x$, $du = -2\,dx$ $\longrightarrow$ $-\frac{1}{2}\int -2e^{-2x}\,dx$

$= -\frac{1}{2}\int e^u\,du$

45. $\int_0^2 (x^2-1)e^{x^3-3x+1}\,dx$

47. $-\int \frac{-e^{-x}}{(1+e^{-x})^2}\,dx$ $u = 1+e^{-x}$, $du = -e^{-x}\,dx$

49. $\int xe^{ax^2}\,dx$

45) $u = x^3 - 3x + 1$

$du = (3x^2-3)\,dx$

$3(x^2-1)\,dx$

51. $\int_1^3 \frac{e^{3/x}}{x^2}\,dx$ $\longrightarrow$ $u = \frac{3}{x} = 3x^{-1}$

$du = -3x^{-2}\,dx$

53. $\int e^{-x}(1 + e^{-x})^2\,dx$

55. $\int e^x\sqrt{1 - e^x}\,dx$

57. $\int \frac{e^x + e^{-x}}{\sqrt{e^x - e^{-x}}}\,dx$ $\longrightarrow$ $u = e^x - e^{-x}$

$du =$

59. $\int \frac{5 - e^x}{e^{2x}}\,dx$

61. $\int_{-2}^0 (3^3 - 5^2)\,dx$

57. $\int \frac{e^x + e^{-x}}{\sqrt{e^x - e^{-x}}}\,dx$

59. $\int \frac{5 - e^x}{e^{2x}}\,dx = \int e^{-2x}(5 - e^x)\,dx$

61. $\int_{-2}^{0} (3^3 - 5^2)\,dx$

$= \int (5e^{-2x} - e^{-x})\,dx$

$= -\frac{5}{2}\int -2e^{-2x}\,dx + \int -e^{-x}\,dx$

let $u = -2x$

$du = -2\,dx$

$= -\frac{5}{2}\int e^u\,du + e^{-x} + C$

$= -\frac{5}{2}e^{-2x} + e^{-x} + C$

In Exercises 63 and 64, find a function f that satisfies the given conditions.

63. $f''(x) = \frac{1}{2}(e^x + e^{-x})$

$f(0) = 1$, $\boxed{f'(0) = 0}$

$f'(x) = \int f''(x)\,dx$

$= \int \frac{1}{2}(e^x + e^{-x})\,dx$

$0 = \frac{1}{2}(e^0 - e^0) + C$

$C = 0$

$f(x) = \int \frac{1}{2}(e^x - e^{-x})\,dx$

$f($

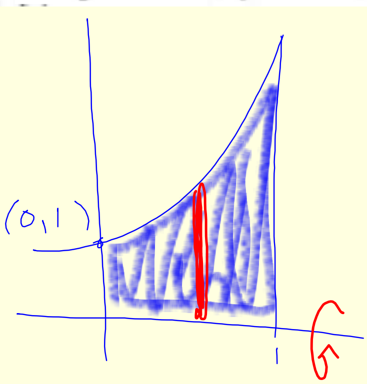
In Exercises 65–68, find the area of the region bounded by the graphs of the given equations.

65. $y = e^x, y = 0, x = 0, x = 5$

67. $y = xe^{-(x^2/2)}, y = 0, x = 0, x = \sqrt{2}$

In Exercises 69 and 70, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the x-axis.

69. $y = e^x, y = 0, x = 0, x = 1$



$$V = \frac{\pi}{2}\int 2(e^x)^2\,dx$$

$$\frac{\pi}{2}\left[e^{2x}\right]_0^1$$

$$\frac{\pi}{2}($$

7.6

The log rule for integration

General Power Rule for Integration

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad \text{for } n \neq -1$$

$$\int x^{-1}\,dx \Rightarrow \frac{x^{-1+1}}{-1+1}$$

Using differentiation of logs and the Fundamental Theorem:

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x} \quad \text{and} \quad \frac{d}{dx}[\ln|u|] = \frac{1}{u}\,du$$

Using differentiation of Logs and the Fundamental Theorem:

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x} \quad \text{and} \quad \frac{d}{dx}[\ln|u|] = \frac{1}{u}du$$

Log Rule for Integration

Let u be a differentiable function of x.

1. $\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\int \frac{1}{u}\,du = \ln|u| + C$

Examples:

$$\int \frac{2}{x}\,dx$$

$$2\int \frac{1}{x}\,dx$$

$$= 2\ln|x| + C$$

$$= \ln x^2 + C$$

$$\frac{1}{2}\int \frac{1\cdot 2}{2x-1}\,dx.$$

$u = 2x-1$

$du = 2\,dx$

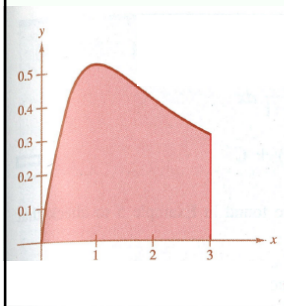
$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$\frac{1}{2}\ln|u| + C$$

$$\frac{1}{2}\ln|2x-1| + C$$

Find the area of the region bounded by:

$y = \dfrac{x}{x^2+1}$, x = 3 and the x-axis



$A = \frac{1}{2}\int_0^3 \frac{2x}{x^2+1}\,dx$ $u = x^2+1$ $du = 2x\,dx$

$A = \frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|x^2+1|\Big]_0^3$

$\frac{1}{2}(\ln(9+1) - \ln(0+1))$

$\frac{1}{2}(\ln 10 - 0)$

$\frac{\ln 10}{2} \approx$

or $\ln\sqrt{10} \approx$

Recognizing quotient forms of the Log Rule

(a) $\displaystyle\int \frac{3x^2+1}{x^3+x}\,dx$

(b) $\displaystyle\frac{1}{2}\int \frac{2(x+1)}{x^2+2x}\,dx$

$u = x^3 + x$

$du = 3x^2 + 1$

$\int \frac{1}{u}\,du$

$\ln|x^3+x| + C$

$u = x^2 + 2x$

$du = 2x + 2$

$= \frac{1}{2}\ln|x^2+2x| + C$

Using long division before integrating

$$\int \frac{x^2 + x + 1}{x^2 + 1}\,dx$$

Rename the integrand:

$$\begin{array}{r} 1 \\ x^2+1 \overline{\big) x^2 + x + 1} \\ -(x^2 \quad + 1) \\ \hline x \end{array} \leftarrow \text{remainder}$$

$$\int \left(1 + \frac{x}{x^2+1}\right) dx$$

$$\int 1\,dx + \int \frac{x}{x^2+1}\,dx \quad \leftarrow \quad u = x^2+1, \quad du = 2x\,dx$$

$$x + \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$$

$$x + \frac{1}{2}\int \frac{1}{u}\,du$$

$$\boxed{x + \frac{1}{2}\ln(x^2+1) + C}$$

Change of variables with the Log Rule

$$\int \frac{2x}{(x+1)^2}\,dx.$$

$u = x+1 \rightarrow x = u-1$

$du = 1dx$

$$2\int \frac{u-1}{u^2}\,du$$

$$2\left[\int \frac{1}{u}\,du - \int u^{-2}\,du\right.$$

General Guidelines for Integration

1. Memorize basic integration formulas.
   Power Rule, Exponent Rule and Log Rule
   (We have 3 so far. There will be 19!)

2. Pick the formula that fits your integrand.
   (You may need to fuss with your choice of u.)

3. If u-sub doesn't work, use algebra to alter your integrand first!

Speaking of fussing...

$$\int \frac{1}{x \ln x}\, dx.$$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$\int \left(\frac{1}{x} \cdot \frac{1}{\ln x}\right) dx$$

$$\int \frac{1}{u}\, du \to \ln|u| + C$$

$$\ln|\ln x| + C$$

Another...

$$\int \frac{1}{\sqrt{x}+1}\,dx.$$

let $u = \sqrt{x} + 1$

$du = \frac{1}{2\sqrt{x}}\,dx$

$$\int \frac{1}{u}\,dx \qquad dx = 2\sqrt{x}\,du$$

$$\int \frac{1}{u}\cdot 2\sqrt{x}\,du \qquad \sqrt{x} = u-1$$

$$2\int \frac{1}{u}(u-1)\,du$$

$$= 2\int \left(\frac{u}{u} - \frac{1}{u}\right)du$$

$$= 2\int 1\,du - 2\int \frac{1}{u}\,du$$

$$= 2(\sqrt{x}+1) - 2\ln(\sqrt{x}+1) + C$$

$$= 2\sqrt{x} - \ln(\sqrt{x}+1)^2 + C$$

Remember differentiation rule:

$$\frac{d}{dx}\left[a^u\right] = (\ln a)\, a^u\, du$$

We need (ln a) in the integrand:

$$\int a^u\,du = \frac{1}{\ln a}\int (\ln a)\, a^u\, du$$

$$= \frac{1}{\ln a}a^u + C$$

$$\frac{a^u}{\ln a} + C$$

$$\int a^x\,dx = \left(\frac{1}{\ln a}\right) a^x + C$$

$$\int 2^x\,dx = \frac{1}{\ln 2}\cdot 2^x + c$$

$$\frac{2^x}{\ln 2} + C$$

HW:

p. 399 # 1 - 33 odd

Week 7 HW quiz tomorrow:

pgs. 342, 354, yellow WS and p. 370